10-5
Base e and the Natural Log

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

n	e
1	2
10	$\left(1+\frac{1}{10}\right)^{10} = 2.5937$
100	$\left(1+\frac{1}{100}\right)^{100} = 2.7048$
1000	2.7169
10,000	2.7181
	2.7183

$e = 2.7183$

$\log_e x = \ln x$

The Natural Log

Exponential Form	Logarithmic Form
$e^{2.64} = 14$	$\ln 14 = 2.64$
$e^{3.09} = 22$	$\ln 22 = 3.09$
$e^{-1.1} = 1/3$	$\ln \frac{1}{3} = -1.1$
$e^{\frac{1}{5}} = 1.22$	$\ln 1.22 = 1/5$
$e^4 = 54.6$	$\ln 54.6 = 4$

Simplify:

$\ln e^4 = 4$

$\ln e^2 = 2$

$\ln \frac{1}{e^3} = -3$

Simplify:

$\ln 1 = 0$

$\ln e^0$

$\ln \sqrt{e} = \frac{1}{2}$

$\ln e = 1$

Simplify:

$e^{\ln 17}$ $= 17$

$e^{\ln 21}$ $= 21$

$\ln e^{4x+3}$ $= 4x+3$

Solve

$\ln 3x = 2$

$\frac{e^2}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$2.4630 = x$

Solve

$\ln (x-5) = 4$

$e^4 = x - 5$

$e^4 + 5 = x$

$59.5982 = x$

Solve

$\ln 2x + \ln x = \ln 8$

$\ln (2x^2) = \ln 8$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$x = 2$

Solve

$e^{4x} = 24$

$\ln e^{4x} = \ln 24$

$4x = \ln 24$

$x = .7945$

If interest is compounded continuously, use the formula:

$A = Pe^{rt}$

A = amount
P = principal
r = rate
t = time

$A = Pe^{rt}$

If $1,000 is compounded continuously at 6% interest:

- How much money would there be in one year?
- How much money would there be in 8 years?

$A = 1000e^{.06(1)}$

$A = \$1061.84$

$A = 1000e^{.06 \cdot 8}$

$\$1616.07$

How long would it take that same principal to reach at least $1350.

$A = Pe^{rt}$

$1350 = 1000\,e^{.06t}$

$1.35 = e^{.06t}$

$\ln 1.35 = \ln e^{.06t}$

$\ln 1.35 = .06t$

$5\text{yrs} \approx t$

HW
p558
30-51 x3, 28, 54, 58, 59

More About...

Money

To determine the doubling time on an account paying an interest rate r that is compounded *annually*, investors use the "Rule of 72." Thus, the amount of time needed for the money in an account paying 6% interest compounded annually to double is $\frac{72}{6}$ or 12 years.

Source: www.datachimp.com

Evaluate each expression.

34. $e^{\ln 0.2}$ **35.** $e^{\ln y}$ **36.** $\ln e^{-4x}$ **37.** $\ln e^{45}$

Solve each equation or inequality.

38. $3e^x + 1 = 5$ **39.** $2e^x - 1 = 0$ **40.** $e^x < 4.5$

41. $e^x > 1.6$ **42.** $-3e^{4x} + 11 = 2$ **43.** $8 + 3e^{3x} = 26$

44. $e^{5x} \geq 25$ **45.** $e^{-2x} \leq 7$ **46.** $\ln 2x = 4$

47. $\ln 3x = 5$ **48.** $\ln (x + 1) = 1$ **49.** $\ln (x - 7) = 2$

50. $\ln x + \ln 3x = 12$ **51.** $\ln 4x + \ln x = 9$

52. $\ln (x^2 + 12) = \ln x + \ln 8$ **53.** $\ln x + \ln (x + 4) = \ln 5$

MONEY For Exercises 54–57, use the formula for continuously compounded interest found in Example 6.

54. If you deposit $100 in an account paying 3.5% interest compounded continuously, how long will it take for your money to double?

55. Suppose you deposit A dollars in an account paying an interest rate r as a percent, compounded continuously. Write an equation giving the time t needed for your money to double, or the *doubling time*.

56. Explain why the equation you found in Exercise 55 might be referred to as the "Rule of 70."